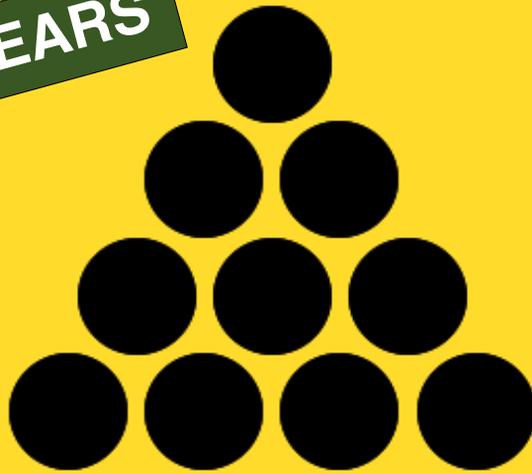


**20 YEARS**



**Program  
Mathe 2000**

**A Unified Approach to  
Learning Mathematics**

## 20 Years Mathe 2000

Gerhard N. Müller and Erich Ch. Wittmann – Some of the most significant contributions to the field of didactics of mathematics are due to these two scholars. The special mark of their work is a conscious reference to the subject matter of teaching.

Mathematics is part of our daily lives and should therefore not be seen by students as an isolated activity. Instead, it should be experienced as a rich body of patterns that find application in our everyday life, such as in football scores or price lists. And rather than being passive recipients of mathematical knowledge, students should be given the opportunity to explore the field on their own – within the framework of active search, such as is provided by **Mathe 2000**.

Counters, posters and patterns of dots blur the lines between informal arguments and scientific proof. The students assimilate just a few basic ideas and in elaborating on them they arrive at more and more complex and sometimes very individual networks of knowledge. Teachers are no longer forced to pace students and to put them off until a future lesson; rather, they are now able to encourage and guide them and, thereby, stimulate and support complete thought processes.

During the last two decades, **Mathe 2000** has greatly influenced the professional discourse on the didactics of mathematics, and recently, in the light of educational principles and standards, it has even gained in practical significance. Yet, the concept's future still lies ahead of us.

A handwritten signature in black ink, appearing to read 'Karl Slipek', written in a cursive style.

Karl Slipek  
*Managing Director at Ernst Klett Verlag GmbH*

## 20 Years Mathe 2000

Gerhard N. Müller and Erich Ch. Wittmann – Some of the most significant contributions to the field of didactics of mathematics are due to these two scholars. The special mark of their work is a conscious reference to the subject matter of teaching.

Mathematics is part of our daily lives and should therefore not be seen by students as an isolated activity. Instead, it should be experienced as a rich body of patterns that find application in our everyday life, such as in football scores or price lists. And rather than being passive recipients of mathematical knowledge, students should be given the opportunity to explore the field on their own – within the framework of active search, such as is provided by **Mathe 2000**.

Counters, posters and patterns of dots blur the lines between informal arguments and scientific proof. The students assimilate just a few basic ideas and in elaborating on them they arrive at more and more complex and sometimes very individual networks of knowledge. Teachers are no longer forced to pace students and to put them off until a future lesson; rather, they are now able to encourage and guide them and, thereby, stimulate and support complete thought processes.

During the last two decades, **Mathe 2000** has greatly influenced the professional discourse on the didactics of mathematics, and recently, in the light of educational principles and standards, it has even gained in practical significance. Yet, the concept's future still lies ahead of us.

A handwritten signature in black ink, appearing to read 'Karl Slipek', written in a cursive style.

Karl Slipek  
*Managing Director at Ernst Klett Verlag GmbH*

# Aim of the Project

First enthusiasm, then diligence.

Stefan Zweig

In 1985, the State of North Rhine-Westphalia in Germany adopted a new mathematics curriculum for elementary schools that contained three notable innovations:

1. Active discovery and inquiry as the main principle of teaching and learning.
2. General mathematical objectives (process objectives) in addition to content objectives
3. An orientation towards mathematical structure in addition to the traditional orientation towards practical applications.

In the following years, this “curriculum of the century” influenced the curriculum development for all grades and also contributed to reshaping educational standards.

We founded **Mathe 2000** as a developmental research project in 1987 in order to support the implementation of the new curriculum, both conceptually and practically. The decision was motivated by our firm belief that the innovations of the new framework were educationally sound and for that reason would be successful.

For the first 15 years we focused our attention on elementary schools and during the last five years on early math education in kindergartens. The framework, however, is designed to extend across all grade levels. The aim is a universal concept of mathematics learning that ranges from kindergarten all the way to the German „Abitur“ and ensures that there are no learning gaps in the course of the students’ education.

Without the support of so many, **Mathe 2000** would have never achieved its current state of development. We would like to extend our sincerest thanks to everyone whose involvement in their respective fields contributed to the program’s success.



Gerhard N. Müller

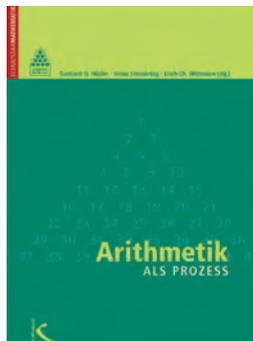


Erich Ch. Wittmann

Co-directors of the **Mathe 2000** program



The **Little Book of Numbers** in the early math program is the entry level of the **mathe 2000** program.



**Arithmetic as a Process**, the first volume in the scholarly series for teacher education, is at the high end of the spectrum.



**Arithmetic With Children**  
 ("Mit Kindern rechnen")  
 Volume 96 in a series published by the *Elementary School Association* ("Grundschulverband") contains reports on teaching experiments with **Mathe 2000** learning environments.



**Beyond PISA. Educational Reform as a Reform of Teaching and Learning. A Five-Point Program from a Systemic Perspective** provides a brief account of the project group's views on education policy.

## The Secret of Success

There is nothing as practical as a good theory.

*Leonardo da Vinci*

The large success of **Mathe 2000** is primarily based on the effective exchange between theory and practice, which has been an integral part of the project work. At the outset of the project, we deliberately placed the design, the empirical research, and the implementation of "substantial learning environments" representing the new syllabus at the very center. The replacement of the once commonly used term "teaching unit" by "learning environment" indicates the change in thinking about teaching and learning as indicated in the "curriculum of the century."

A learning environment is composed of goals, technical and didactical principles, working materials, possible sequences of activities (tasks) as well as provisions for quality control. Thus, teachers are provided with concrete information for teaching specific topics in class or in kindergarten. Any required background knowledge is included. *The Handbook of Productive Arithmetic Exercises* ("Handbuch produktiver Rechenübungen") is a perfect model for this type of practice-oriented developmental research.

The **Mathe 2000** approach is attractive to teachers also because it offers them various choices to creatively orchestrate learning environments and because it stimulates collaboration between teachers. As a result, a climate of innovation emerged that greatly benefited our project. Much of the experience gained during the implementation of the **Mathe 2000** learning environments flowed back into the project and gave way to important impulses for further work. The hub of the "theory and practice network" is the annual Symposium „mathe 2000“. With 450 attendees, the symposium has been fully booked for the last 15 years.

# Principles

Ideas are more important than recipes.

*Ferran Adrià, 2006 "The best chef in the world"*

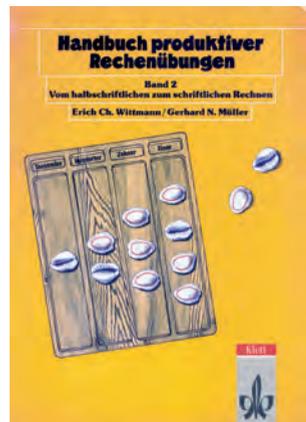
**Mathe 2000** offers a coherent overall concept for teaching and learning mathematics. The emphasis is on mathematical understanding and on a clear structure of the mathematics curriculum. Mathematics is viewed as the science of interesting and useful patterns and structures that can be actively or interactively acquired and applied through playful inquiry. Patterns and structures are not just objects that offer resistance to young learners; rather, they are also the perfect aids for understanding mathematics. To ensure that these aids can be used, it is essential to continually develop the content by focusing on the essentials. The fundamental concepts must be introduced at an early stage and gradually expanded over the various levels. For example, the treatment of multiplication is based on the laws of arithmetic already in the second grade, and this algebraic approach is pursued systematically up to secondary level.

Within the **Mathe 2000** learning environments, patterns and structures provide options for learning so that learners with varying levels of competence are able to engage according to their individual abilities ("natural differentiation"). This explains why the **Mathe 2000** concept is particularly successful in working with slow learners and learning-disabled children. There are also many opportunities for fast learners who want to explore and further immerse themselves into the patterns and structures.

Practicing skills is of central importance in the project as it is considered crucial to the success of learning in the long run. The range varies from productive to automated exercises. Because productive exercises are based on patterns and structures, optimal conditions for real self-monitoring are given.



The **Handbook of Productive Arithmetic Exercises** in two volumes is the centerpiece of **Mathe 2000**. The first volume describes learning environments that foster the learning of addition and multiplication; the second volume describes informal calculation strategies and standard algorithms.



## The Early Math Program

This: Thinking actively

This: Inspiring active thinking

is the source of all productive education.

*Friedrich Fröbel, The Education of Man 1826*



The games in both volumes of **The Little Book of Numbers** develop the number concept and its many facets. The children also learn about number patterns at the same time.



“Playful” access to mathematics is meant literally in the **Mathe 2000** early math program. The learning environments consist of mathematical games and instructions for making geometric models. All are based on sound mathematical principles. As a result of play and building activities, manipulations and thought processes are repeated in multiple variations. The children obtain continuous feedback on their experiments so that they are able to make improvements each time. The entry thresholds are low enough for small children to join in on the games and to learn from the bigger ones.

From the very beginning, the program has been based on the motive force of mathematics itself. Exploring relationships between numbers or geometric forms, finding solutions for problems, making models by following instructions, playing games by following specific rules: all of this stimulates children and is consistent with their playful inclinations. We reject the idea of packaging mathematics into fun disguises (“edutainment”) because that would put children on the wrong track. You learn mathematics with mathematics, not with pseudo-mathematics. Sensitizing teachers, parents, and children to a “no frills” and truly playful approach to real mathematics is the conscious goal of our project work.

One format of the early math program consists of five boxes with games and building instructions.

Part 1 of *The Little Book of Numbers* (“Das kleine Zahlenbuch”) furthers the knowledge of the number concept and its many facets (quantity, ordinal numbers, counting numbers, numerical values, codes).

In Part 2, the children learn to decompose sets into distinct groups and add up quantities of groups instead

of simply counting individual elements one by one. This ability is essential for clever calculations.

Both parts of *The Little Book of Forms* (“Das kleine Formenbuch”) develop the coordination of the brain, hands, and eyes in various ways. Part 1 deals with the comprehension and examination of forms through hands-on play. The children fit building blocks together, manipulate forms with mirrors, and move through labyrinths. An intense training of the fine motor skills is devised through the learning environments in Part 2. The children grasp geometric shapes and figures (sphere, cylinder, cube) by means of tangible objects as they fold, knead, draw, and build.

*The Little Book of Logical Games* (“Das kleine Denkspielbuch”) encourages rule-guided play, the classification of objects according to their properties and relationships, logical thinking, and problem solving.

The **Mathe 2000** early math program deliberately targets all relevant content areas and all general mathematical competencies as outlined in the educational standards for mathematics teaching in elementary schools. The program, therefore, ensures a smooth transition from early math to primary mathematics teaching and the textbook *The Book of Numbers* (“Das Zahlenbuch”).

A second, more compact format of the early math program, *The Book of Numbers* for the kindergarten, is in preparation. Like *The Book of Numbers* for primary school, the new format will consist of a book with games and instructions for making geometric models (2 parts), a workbook (2 parts) and a handbook. This new format will appear in 2009.



Both volumes of the **The Little Book of Forms** were inspired by Friedrich Fröbel’s games. The focal point here is the active exploration of forms.



In **The Little Book of Logical Games** the concept of mathematics as a science of patterns and structures is particularly evident.

## The Primary Program

A young child is an active being that must be encouraged in its spontaneous search of knowledge.

Jean Piaget



Long before TIMSS and PISA, **The Book of Numbers** set the standards in connecting proven traditions with innovations in teaching and learning. It puts the same emphasis on the basics and on exploring patterns.

Since the beginning of the nineties, the *Handbook of Productive Arithmetic Exercises* has inspired many teachers to conduct teaching experiments based on active discovery and inquiry. As these experiments were successful, there was a strong demand for a textbook consistent with this approach. The directors of the project entered an agreement with the *Ernst Klett Verlag* under the condition they obtain complete freedom in developing a new type of textbook. To minimize the considerable risk as perceived by the publisher at the time, it was agreed upon to first launch a trial run of all four volumes of the new book in 40 German and 20 Swiss classes for the duration of four years. Page by page the teachers involved evaluated the student textbooks and workbooks against their experiences in the classroom. The results, which revealed surprisingly strong agreement among the teachers, provided relevant data for small revisions. Overall, the trial run was very successful, and so the project was given green light.

*The Book of Numbers* ("Das Zahlenbuch") was published volume by volume during 1994-1997 and immediately received a tremendous response. The number of teachers who were familiarizing themselves with the concept and using *The Book of Numbers* because of their conviction to the program rose steadily in the years to follow. This trend intensified again in 2004-2007 when the revised edition was published. That *The Book of Numbers* served as a model for other books and was adapted for other countries, most notably for Switzerland, underscores the importance of the book in the development of mathematics education beyond borders.

In addition to *The Book of Numbers*, there are other materials available within the elementary program – primarily the “basic courses” on numbers, forms, and quantities as well as the CD-ROM *Calculightning* (“Blitzrechnen”). These materials rest on the conviction that in all content areas there are basic skills that require intensive practice. The CD-ROM “*Calculightning*” supports quality control in two ways: It contains modules that allow children to monitor themselves and an “assessment tool” that allows teachers to monitor the children’s progress.

Geometric and logical thinking are enhanced by materials for individualized use: two *Mirror Books* (“Spiegelbücher”), the two volumes of *Looking and Building* (“Schauen und Bauen”) as well as the two-volume *School of Logical Thinking* (“Die Denkschule”). The **Mathe 2000** program is complemented by working materials that fit into the overall concept, for example, the *Thousand Book* (“Das Tausenderbuch”) and the *Million Book* (“Das Millionenbuch”) – materials that assist students in advancing through the decimal number system.

In its present form *The Book of Numbers* along with the accompanying materials is a coherent and fully developed textbook that meets all requirements set by educational standards, and it does so in a distinctive fashion.

Because of its philosophy, *The Book of Numbers* is also well-suited for use in combined classes. In particular, it maintains a consistent structure of mathematics teaching across the grades, which is essential for an organizational form such as this – a point which is unfortunately often overlooked.



Part 1 of **Looking and Building** offers geometric games with cuboids. Part 2 of the set is **Solving Soma Cube Puzzles** (“Spiele mit dem Somawürfel”). Both books teach how to think in terms of spatial relationships and offer an introduction to the fundamentals of descriptive geometry. Moreover, they support social learning in a distinctive way.





The two parts of the card file **Simple Word Problems. Basic Course in Quantities** provide learners with opportunities to practice basic competencies in applying mathematics.



With its card file **Mental Geometry, Basic Course in Forms** that extends into secondary schools, **Mathe 2000** charts new territory.

## Perspectives

One must explore, develop, reform the underbelly of the New and the Old but neither revolutionize forwards nor backwards.

*K. G. von Raumer*

The developmental work for primary schools and early math education has been concluded for the most part. In collaboration with teachers and educators, the focus of the project work is now to enhance the effective use of materials. Three areas are of particular importance:

### Revision of the *Handbook of Productive Arithmetic Exercises*

Since the first publication of the “Handbook” 20 years have passed. So more than a face-lift is necessary. The new handbook will include a broader explication of the mathematical background and will also contain suggestions for the orchestration of learning environments. **“Calculightning Initiative”**

The success of *The Book of Numbers* depends largely on the consistent follow-through of the basic courses, especially the *Calculightning* program. This requires a new way of thinking about the significance of basic competencies and new organizational forms that support the practice of skills.

#### ► Interface between Kindergarten and Elementary School

Pilot projects must be conducted to ascertain how the **Mathe 2000** early math program can be organically integrated into the work in kindergartens, how primary schools can build on it, and how it can be continued for particular groups of children before they enter into systematic education.

As far as secondary education is concerned, *The Book of Numbers* will be continued in grades 5 and 6. In the project directors' view, these grades are part of primary school – as is common in almost all other European countries.

**The faster a thing is created,  
the more fleeting its permanence,  
the slower, the more enduring it will be.**

*Friedensreich Hundertwasser*