

Counters & co. digitally. A universal software for primary mathematics.

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The systematic use of epistemologically well founded standard representations of basic arithmetical and geometric structures together with the operative principle has been one of the pillars on which the developmental research conducted in the project Mathe 2000 over the past 30 years has been based.

The new software contains 6x6 modules with digital versions of these basic representations. Its main objective is to reinforce the **teacher-student interaction** in introducing students to an activity and in discussing the results. This interaction is seen as crucial for understanding mathematics.

During the poster session ICME-participants are invited to explore this software “hands one” and to discuss its conception with the authors. The following descriptions of three modules are intended to give an impression of the “flavor” of this software.

Module “Dot arrays”

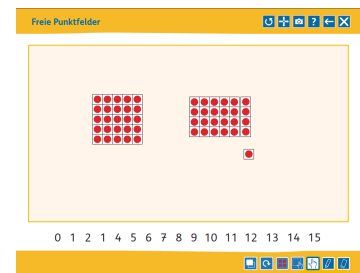
Rectangular arrays of dots are the only means of representation from which the arithmetical laws of multiplication can be derived in a way that is accessible to primary students. Therefore they play a crucial role in learning the multiplication table. The module provides digital versions of dot arrays that allow for operative proofs.

Example:

While calculating the following series of multiplication problems the students discover that the results in the first and second line differ by 1:

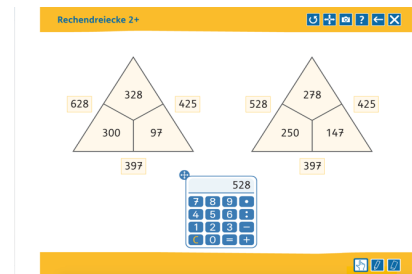
$$\begin{array}{r} 1 \cdot 1 = \quad 2 \cdot 2 = \quad 3 \cdot 3 = \quad 4 \cdot 4 = \quad 5 \cdot 5 = \quad 6 \cdot 6 = \quad 7 \cdot 7 = \quad 8 \cdot 8 = \quad 9 \cdot 9 = \quad 10 \cdot 10 = \\ 0 \cdot 2 = \quad 0 \cdot 3 = \quad 2 \cdot 4 = \quad 3 \cdot 5 = \quad 4 \cdot 6 = \quad 5 \cdot 7 = \quad 6 \cdot 8 = \quad 7 \cdot 9 = \quad 8 \cdot 10 = \quad 9 \cdot 11 = \end{array}$$

With the assistance offered by the teacher this pattern is explained by means of arrays of dots: Any square array can be transformed into an array with one row less and one column more, whereby one dot is recognized as “superfluous”.



Module “Arithmogons”

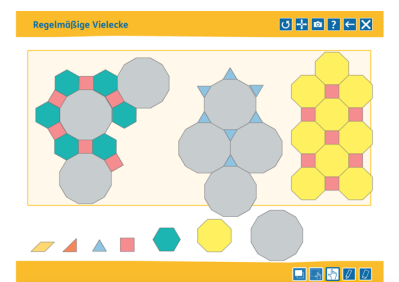
The screen shows two empty arithmogons. The fields inside and outside can be filled with numbers that can be drawn from the display of an integrated hand held calculator. The availability of two arithmogons allows for operative variations and for the comparisons of the results. The most difficult type of problems, the determination of the numbers inside when the numbers outside are given can be solved by systematic trials. Again the results found by students with paper and pencil can be demonstrated and commented by means of this digital module.



Module “Regular polygons”

Regular polygons with 3, 4, 5, 6, 8 and 12 vertices and equal sides are available as “endless sources”. The shapes can be pulled on the screen, translated and rotated by 15° by means of a double-click. So it is possible to create tessellations. In particular all 3 regular and 8 semi-regular tessellations can be produced.

Students are provided with a template of these polygons that allows for drawing the tessellations on a sheet of paper before and after working with the digital module.



References

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